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Chapter 10

AIRGAP FIELD SPACE HARMONICS, PARASITIC TORQUES, RADIAL FORCES, AND NOISE

The airgap field distribution in induction machines is influenced by stator and rotor mmf distributions and by magnetic saturation in the stator and rotor teeth and yokes (back cores).

Previous chapters introduced the mmf harmonics but were restricted to the fundamentals. Slot openings were considered but only in a global way, through an apparent increase of airgap by the Carter coefficient.

We first considered magnetic saturation of the main flux path through its influence on the airgap flux density fundamental. Later on a more advanced model was introduced (AIM) to calculate the airgap flux density harmonics due to magnetic saturation of main flux path (especially the third harmonic).

However, as shown later in this chapter, slot leakage saturation, rotor static, and dynamic eccentricity together with slot openings and mmf step harmonics produce a multitude of airgap flux density space harmonics. Their consequences are parasitic torque, radial uncompensated forces, and harmonics core and winding losses. The harmonic losses will be treated in the next chapter.

In what follows we will use gradually complex analytical tools to reveal various airgap flux density harmonics and their parasitic torques and forces. Such treatment is very intuitive but is merely qualitative and leads to rules for a good design. Only FEM–2D and 3D—could depict the extraordinary involved nature of airgap flux distribution in IMs under various factors of influence, to a good precision, but at the expense of much larger computing time and in an intuitiveless way. For refined investigation, FEM is, however, “the way”.

10.1. STATOR MMF PRODUCED AIRGAP FLUX HARMONICS

As already shown in Chapter 4 (Equation 4.27), the stator mmf per-pole-stepped waveform may be decomposed in harmonics as

$$F_l(x, t) = \frac{3W_l I_l \sqrt{2}}{\pi p_l} \left[K_{w1} \cos\left(\frac{\pi}{\tau} x - \omega_l t\right) + \frac{K_{w5}}{5} \cos\left(\frac{5\pi}{\tau} x + \omega_l t\right) + \frac{K_{w7}}{7} \cos\left(\frac{7\pi}{\tau} x - \omega_l t\right) + \frac{K_{w11}}{11} \cos\left(\frac{11\pi}{\tau} x + \omega_l t\right) + \frac{K_{w13}}{13} \cos\left(\frac{13\pi}{\tau} x - \omega_l t\right) \right] \quad (10.1)$$

where K_{wv} is the winding factor for the v^{th} harmonic,

$$K_{wv} = K_{qv} K_{yv}; \quad K_{qv} = \frac{\sin \frac{v\pi}{6}}{q \sin \frac{v\pi}{6q}}; \quad K_{yv} = \sin \frac{v\pi y}{2\tau} \quad (10.2)$$

In the absence of slotting, but allowing for it globally through Carter's coefficient and for magnetic circuit saturation by an equivalent saturation factor K_{sv} , the airgap field distribution is

$$\begin{aligned} B_{gl}(\theta, t) = & \frac{3\mu_0 W_l I_l \sqrt{2}}{\pi p_l g K_c} \left[\frac{K_{w1}}{1 + K_{s1}} \cos(\theta - \omega_l t) + \frac{K_{w5}}{5(1 + K_{s5})} \cos(5\theta + \omega_l t) + \right. \\ & + \frac{K_{w7}}{7(1 + K_{s7})} \cos(7\theta - \omega_l t) + \frac{K_{w11}}{11(1 + K_{s11})} \cos(11\theta + \omega_l t) + \\ & \left. + \frac{K_{w13}}{13(1 + K_{s13})} \cos(13\theta - \omega_l t) \right]; \quad \theta = \frac{\pi}{\tau} x \end{aligned} \quad (10.3)$$

In general, the magnetic field path length in iron is shorter as the harmonics order gets higher (or its wavelength gets smaller). K_{sv} is expected to decrease with v increasing.

Also, as already shown in Chapter 4, but easy to check through (10.2) for all harmonics of the order v ,

$$v = C_1 \frac{N_s}{p_1} \pm 1 \quad (10.4)$$

the distribution factor is the same as for the fundamental.

For three-phase symmetrical windings (with integer q slots/pole/phase), even order harmonics are zero and multiples of three harmonics are zero for star connection of phases. So, in fact,

$$v = 6C_1 \pm 1 \quad (10.5)$$

As shown in Chapter 4 (Equations 4.17 – 4.19) harmonics of 5th, 11th, 17th, ... order travel backwards and those of 7th, 13th, 19th, ... order travel forwards – see Equation (10.1).

The synchronous speed of these harmonics ω_v is

$$\omega_v = \frac{d\theta_v}{dt} = \frac{\omega_l}{v} \quad (10.6)$$

In a similar way, we may calculate the mmf and the airgap field flux density of a cage winding.

10.2. AIRGAP FIELD OF A SQUIRREL CAGE WINDING

A symmetric (healthy) squirrel cage winding may be replaced by an equivalent multiphase winding with N_r phases, $\frac{1}{2}$ turns/phase and unity winding factor. In this case, its airgap flux density is

$$B_{g2}(\theta, t) = \frac{N_r \mu_0 I_1 \sqrt{2}}{\pi p_1 g K_c} \sum_{\mu=1}^{\infty} \frac{\cos(\mu\theta \mp \omega_1 t - \phi_{12v})}{\mu(1 + K_{sv})} = \frac{\mu_0 F_2(t)}{g K_c} \quad (10.7)$$

The harmonics order which produces nonzero mmf amplitudes follows from the applications of expressions of band factors K_{BI} and K_{BII} for $m = N_r$:

$$\mu = C_2 \frac{N_r}{p_1} \pm 1 \quad (10.8)$$

Now we have to consider that, in reality, both stator and rotor mmfs contribute to the magnetic field in the airgap and, if saturation occurs, superposition of effects is not allowed. So either a single saturation coefficient is used (say $K_{sv} = K_{s1}$ for the fundamental) or saturation is neglected ($K_{sv} = 0$).

We have already shown in Chapter 9 that the rotor slot skewing leads to variation of airgap flux density along the axial direction due to uncompensated skewing rotor mmf. While we investigated this latter aspect for the fundamental of mmf, it also applies for the harmonics. Such remarks show that the above analytical results should be considered merely as qualitative.

10.3. AIRGAP CONDUCTANCE HARMONICS

Let us first remember that even the step harmonics of the mmf are due to the placement of windings in infinitely thin slots. However, the slot openings introduce a kind of variation of airgap with position. Consequently, the airgap conductance, considered as only the inverse of the airgap, is

$$\frac{1}{g(\theta)} = f(\theta) \quad (10.9)$$

Therefore the airgap change $\Delta(\theta)$ is

$$\Delta(\theta) = \frac{1}{f(\theta)} - g \quad (10.10)$$

With stator and rotor slotting,

$$g(\theta) = g + \Delta_1(\theta) + \Delta_2(\theta - \theta_r) = \frac{1}{f_1(\theta)} + \frac{1}{f_2(\theta - \theta_r)} - g \quad (10.11)$$

$\Delta_1(\theta)$ and $\Delta_2(\theta - \theta_r)$ represent the influence of stator and rotor slot openings alone on the airgap function.

As $f_1(\theta)$ and $f_2(\theta - \theta_r)$ are periodic functions whose period is the stator (rotor) slot pitch, they may be decomposed in harmonics:

$$f_1(\theta) = a_0 - \sum_{v=1}^{\infty} a_v \cos v N_s \theta$$

$$f_2(\theta - \theta_r) = b_0 - \sum_{v=1}^{\infty} b_v \cos v N_r (\theta - \theta_r)$$
(10.12)

Now, if we use the conformal transformation for airgap field distribution in presence of infinitely deep, separate slots—essentially Carter’s method—we obtain [1]

$$a_v, b_v = \frac{\beta}{g} F_v \left(\frac{b_{os,r}}{t_{s,r}} \right)$$
(10.13)

$$F_v \left(\frac{b_{os,r}}{t_{s,r}} \right) = \frac{1}{v} \frac{4}{\pi} \left[0.5 + \frac{\left(\frac{v b_{os,r}}{t_{s,r}} \right)^2}{0.18 - 2 \left(\frac{v b_{os,r}}{t_{s,r}} \right)^2} \right] \sin \left(1.6 \frac{\pi v b_{os,r}}{t_{s,r}} \right)$$
(10.14)

β is [2]

Table 10.1 $\beta(b_{os,r}/g)$

$b_{os,r}/g$	0	0.5	1.0	1.5	2.0	3.0	4.0	5.0
β	0.0	0.0149	0.0528	0.1	0.1464	0.2226	0.2764	0.3143
$b_{os,r}/g$	6.0	7.0	8.0	10.0	12.0	40.0	∞	
β	0.3419	0.3626	0.3787	0.4019	0.4179	0.4750	0.5	

Equations (10.13 – 10.14) are valid for $v = 1, 2, \dots$. On the other hand, as expected,

$$a_0 \approx \frac{1}{K_{c1} g} \quad b_0 \approx \frac{1}{K_{c2} g}$$
(10.15)

where K_{c1} and K_{c2} are Carter’s coefficients for the stator and rotor slotting, respectively, acting separately.

Finally, with a good approximation, the inversed airgap function $1/g(\theta, \theta_r)$ is

$$\lambda_g(\theta, \theta_r) = \frac{1}{g(\theta, \theta_r)} \approx \frac{1}{g} \left\{ \frac{1}{K_{c1} K_{c2}} - \frac{a_1}{K_{c2}} \cos N_s \frac{\theta}{p_1} - \frac{b_1}{K_{c1}} \cos N_r \frac{(\theta - \theta_r)}{p_1} + \right.$$

$$\left. + a_1 b_1 \left[\cos \left(\frac{(N_s + N_r)}{p_1} \theta - \frac{N_r}{p_1} \theta_r \right) + \cos \left(\frac{(N_s - N_r)}{p_1} \theta + \frac{N_r}{p_1} \theta_r \right) \right] \right\}$$
(10.16)

As expected, the average value of $\lambda_g(\theta, \theta_r)$ is

$$(\lambda_g(\theta, \theta_r))_{\text{average}} = \frac{1}{K_{c1}K_{c2}g} = \frac{1}{K_{cg}} \quad (10.17)$$

θ, θ_r – electrical angles.

We should notice that the inversed airgap function (or airgap conductance) λ_g has harmonics related directly to the number of stator and rotor slots and their geometry.

10.4. LEAKAGE SATURATION INFLUENCE ON AIRGAP CONDUCTANCE

As discussed in Chapter 9, for semiopen or semiclosed stator (rotor) slots at high currents in the rotor (and stator), the teeth heads get saturated. To account for this, the slot openings are increased. Considering a sinusoidal stator mmf and only stator slotting, the slot opening increased by leakage saturation b_{os}' varies with position, being maximum when the mmf is maximum (Figure 10.1).

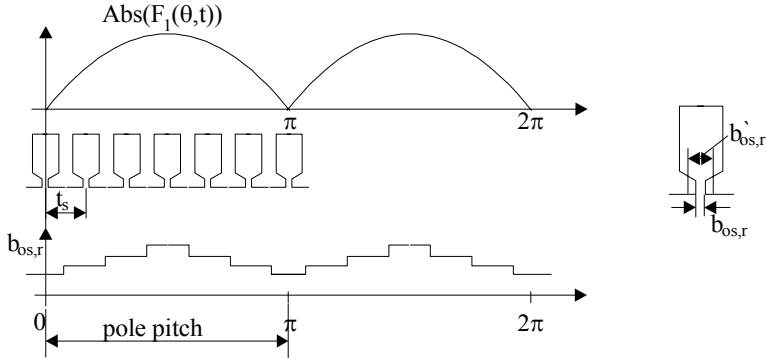


Figure 10.1 Slot opening $b_{os,r}$ variation due to slot leakage saturation

We might extract the fundamental of $b_{os,r}(\theta)$ function:

$$b'_{os,r} \approx b_{os,r}^0 - b''_{os,r} \cos(2\theta - \omega_1 t); \quad \theta - \text{electric angle} \quad (10.18)$$

The leakage slot saturation introduces a $2p_1$ pole pair harmonic in the airgap permeance. This is translated into a variation of a_0, b_0 .

$$a_0, b_0 = a_0^0, b_0^0 - a_0', b_0' \sin(2\theta - \omega_1 t) \quad (10.19)$$

a_0^0, b_0^0 are the new average values of $a_0(\theta)$ and $b_0(\theta)$ with leakage saturation accounted for.

This harmonic, however, travels at synchronous speed of the fundamental wave of mmf. Its influence is notable only at high currents.

Example 10.1. Airgap conductance harmonics

Let us consider only the stator slotting with $b_{os}/t_s = 0.2$ and $b_{os}/g = 4$ which is quite a practical value, and calculate the airgap conductance harmonics.

Solution

Equations (10.13 and 10.14) are to be used with β from Table 10.1, $\beta(4) = 0.2764$,

$$\text{with } K_{cl} \approx \frac{t_s}{t_s - 1.6\beta b_{os}} = \frac{1}{1 - 1.6 \cdot 0.2764 \cdot 0.2} = 1.097$$

$$a_0 = \frac{1}{g} \frac{1}{K_{cl}} = \frac{1}{g} \frac{1}{1.097} = \frac{0.91155}{g}$$

$$a_1 = \frac{1}{g} 0.2764 \frac{4}{\pi} \left[0.5 + \frac{(1 \cdot 0.2)^2}{0.78 - 2 \cdot (0.2)^2} \right] \sin(1.6\pi \cdot 1 \cdot 0.2) = \frac{1}{g} 0.1542$$

$$a_2 = \frac{1}{g} 0.2764 \frac{4}{2\pi} \left[0.5 + \frac{(2 \cdot 0.2)^2}{0.78 - 2 \cdot (2 \cdot 0.2)^2} \right] \sin(1.6\pi \cdot 2 \cdot 0.2) = \frac{1}{g} 0.04886$$

The airgap conductance $\lambda_1(\theta)$ is

$$\lambda_1(\theta) = \frac{1}{g(\theta)} = \frac{1}{g} [0.91155 - 0.1542 \cos(N_s \theta) - 0.04886 \cos(2N_s \theta)]$$

If for the v th harmonics the “sine” term in (10.14) is zero, so is the v th airgap conductance harmonic. This happens if

$$1.6\pi v \frac{b_{os,r}}{t_{s,r}} = \pi, \quad v \frac{b_{os,r}}{t_{s,r}} = 0.625$$

Only the first 1, 2 harmonics are considered, so only with open slots may the above condition be approximately met.

10.5. MAIN FLUX SATURATION INFLUENCE ON AIRGAP CONDUCTANCE

In Chapter 9 an iterative analytical model (AIM) to calculate the main flux distribution accounting for magnetic saturation was introduced. Finally, we showed the way to use AIM to derive the airgap flux harmonics due to main flux path (teeth or yokes) saturation. A third harmonic was particularly visible in the airgap flux density.

As expected, this is the result of a virtual second harmonic in the airgap conductance interacting with the mmf fundamental, but it is the resultant (magnetizing) mmf and not only stator (or rotor) mmfs alone.

The two second order airgap conductance harmonics due to leakage slot flux path saturation and main flux path saturation are phase shifted as their originating mmfs are by the angle $\varphi_m - \varphi_1$ for the stator and $\varphi_m - \varphi_2$ for the rotor.

$\varphi_1, \varphi_2, \varphi_m$ are the stator, rotor, magnetization mmf phase shift angle with respect to phase A axis.

$$\lambda_s(\theta_s) = \frac{1}{gK_c} \left[\frac{1}{1 + K_{s1}} + \frac{1}{1 + K_{s2}} \sin(2\theta - \omega_1 t - (\varphi_m - \varphi_1)) \right] \quad (10.20)$$

The saturation coefficients K_{s1} and K_{s2} result from the main airgap field distribution decomposition into first and third harmonics.

There should not be much influence (amplification) between the two second order saturation-caused airgap conductance harmonics unless the rotor is skewed and its rotor currents are large (> 3 to 4 times rated current), when both the skewing rotor mmf and rotor slot mmf are responsible for large main and leakage flux levels, especially toward the axial ends of stator stack.

10.6. THE HARMONICS-RICH AIRGAP FLUX DENSITY

It has been shown [1] that, in general, the airgap flux density $B_g(\theta, t)$ is

$$B_g(\theta, t) = \mu_0 \lambda_{1,2}(\theta) F_v \cos(v\theta \mp \omega t - \varphi_v) \quad (10.21)$$

where F_v is the amplitude of the mmf harmonic considered, and $\lambda(\theta)$ is the inversed airgap (airgap conductance) function. $\lambda(\theta)$ may be considered as containing harmonics due to slot openings, leakage, or main flux path saturation. However, using superposition in the presence of magnetic saturation is not correct in principle, so mere qualitative results are expected by such a method.

10.7. THE ECCENTRICITY INFLUENCE ON AIRGAP MAGNETIC CONDUCTANCE

In rotary machines, the rotor is hardly ever located symmetrically in the airgap either due to the rotor (stator) unroundedness, bearing eccentric support, or shaft bending.

An one-sided magnetic force (uncompensated magnetic pull) is the main result of such a situation. This force tends to increase further the eccentricity, produce vibrations, noise, and increase the critical rotor speed.

When the rotor is positioned off center to the stator bore, according to [Figure 10.2](#), the airgap at angle θ_m is

$$g(\theta_m) = R_s - R_r - e \cos \theta_m = g - e \cos \theta_m \quad (10.22)$$

where g is the average airgap (with zero eccentricity: $e = 0.0$).

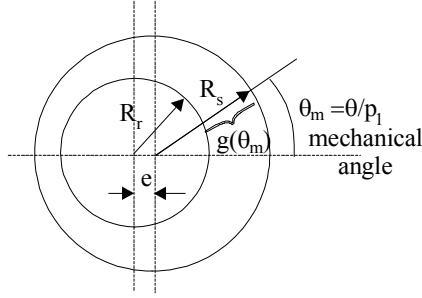


Figure 10.2 Rotor eccentricity R_s – stator radius, R_r – rotor radius

The airgap magnetic conductance $\lambda(\theta_m)$ is

$$\lambda(\theta_m) = \frac{1}{g(\theta_m)} = \frac{1}{g(1 - \varepsilon \cos \theta_m)}; \quad \varepsilon = \frac{e}{g} \quad (10.23)$$

Now (10.23) may be easily decomposed into harmonics to obtain

$$\lambda(\theta_m) = \frac{1}{g} (c_0 + c_1 \cos \theta_m + \dots) \quad (10.24)$$

with

$$c_0 = \frac{1}{\sqrt{1 - \varepsilon^2}}; \quad c_1 = \frac{2(c_0 - 1)}{\varepsilon} \quad (10.25)$$

Only the first geometrical harmonic (notice that the period here is the entire circumference: $\theta_m = \theta/p_1$) is hereby considered.

The eccentricity is static if the angle θ_m is a constant, that is if the rotor revolves around its axis but this axis is shifted with respect to stator axis by e .

In contrast, the eccentricity is dynamic if θ_m is dependent on rotor motion.

$$\theta_m = \theta_m - \frac{\omega_r t}{p_1} = \theta_m - \frac{\omega_1(1-S)t}{p_1} \quad (10.26)$$

It corresponds to the case when the axis of rotor revolution coincides with the stator axis but the rotor axis of symmetry is shifted.

Now using Equation (10.21) to calculate the airgap flux density produced by the mmf fundamental as influenced only by the rotor eccentricity, static and dynamic, we obtain

$$B_g(\theta, t) = \mu_0 F_1 \cos(\theta - \omega_1 t) \frac{1}{g} \left[c_0 + c_1 \cos\left(\frac{\theta}{p_1} - \varphi_s\right) + c_1' \cos\left(\frac{\theta - \omega_1(1-S)t}{p_1} - \varphi_d\right) \right] \quad (10.27)$$

As seen from (10.27) for $p_1 = 1$ (2 pole machines), the eccentricity produces two homopolar flux densities, $B_{gh}(t)$,

$$B_{gh}(t) = \frac{\left[\frac{\mu_0 F_1 c_1}{g} \cos(\omega_1 t - \varphi_s) + \frac{\mu_0 F_1 c_1'}{g} \cos(S\omega_1 t - \varphi_d) \right]}{1 + c_h} \quad (10.28)$$

These homopolar components close their flux lines axially through the stator frame then radially through the end frame, bearings, and axially through the shaft (Figure 10.3).

The factor c_h accounts for the magnetic reluctance of axial path and end frames and may have a strong influence on the homopolar flux. For nonmagnetic frames and (or) insulated bearings c_h is large, while for magnetic steel frames, c_h is smaller. Anyway, c_h should be much larger than unity at least for the static eccentricity component because its depth of penetration (at ω_1), in the frame, bearings, and shaft, is small. For the dynamic component, c_h is expected to be smaller as the depth of penetration in iron (at $S\omega_1$) is larger.

The d.c. homopolar flux may produce a.c. voltage along the shaft length and, consequently, shaft and bearing currents, thus contributing to bearing deterioration.

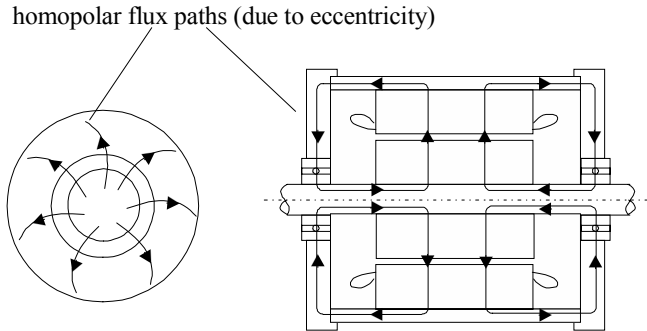


Figure 10.3 Homopolar flux due to rotor eccentricity

10.8. INTERACTIONS OF MMF (OR STEP) HARMONICS AND AIRGAP MAGNETIC CONDUCTANCE HARMONICS

It is now evident that various airgap flux density harmonics may be calculated using (10.21) with the airgap magnetic conductance $\lambda_{1,2}(\theta)$ either from (10.16) to account for slot openings, with a_0 , b_0 from (10.19) for slot leakage saturation, or with $\lambda_s(\theta_s)$ from (10.20) for main flux path saturation, or $\lambda(\theta_m)$ from (10.24) for eccentricity.

$$B_g(x, t) = \frac{\mu_0}{g} \left[F_1(\theta, t) + F_2(\theta, t) \right] \left[\begin{matrix} \lambda_{1,2}(\theta) & + \lambda_s(\theta) & + \lambda\left(\frac{\theta}{p_1}\right) \\ (10.16) \text{ with } (10.19) & (10.20) & (10.24) \end{matrix} \right] \quad (10.29)$$

As expected, there will be a very large number of airgap flux density harmonics and its complete exhibition and analysis is beyond our scope here.

However, we noticed that slot openings produce harmonics whose order is a multiple of the number of slots (10.12). The stator (rotor) mmfs may produce harmonics of the same order either as sourced in the mmf or from the interaction with the first airgap magnetic conductance harmonic (10.16).

Let us consider an example where only the stator slot opening first harmonic is considered in (10.16).

$$B_{gv}(\theta, t) \approx \frac{\mu_0 F_{lv}}{g} \cos(v\theta' - \omega_1 t) \left[\frac{1}{K_{cl}} + a_1 \cos \frac{N_s \theta}{p_1} \right] \quad (10.30)$$

$$\text{with} \quad \theta' = \theta + \frac{\pi}{N_s} p_1 \quad (10.31)$$

θ' takes care of the fact that the axis of airgap magnetic conductance falls in a slot axis for coil chordings of 0, 2, 4 slot pitches. For odd slot pitch coil chordings, $\theta' = \theta$. The first step (mmf) harmonic which might be considered has the order

$$v = c_1 \frac{N_s}{p_1} \pm 1 = \frac{N_s}{p_1} \pm 1 \quad (10.32)$$

Writing (10.30) into the form

$$B_{gv}(\theta, t) = \frac{\mu_0 F_{lv}}{g K_{cl}} \cos \left[v \left(\theta + \frac{\pi p_1}{N_s} \right) - \omega_1 t \right] + \frac{a_1 \mu_0 F_{lv}}{2} \cos \left[v \left(\theta + \frac{\pi p_1}{N_s} \right) + \frac{N_s \theta}{p_1} - \omega_1 t \right] + \cos \left[v \left(\theta + \frac{\pi p_1}{N_s} \right) - \frac{N_s \theta}{p_1} - \omega_1 t \right] \quad (10.33)$$

For $v = \frac{N_s}{p_1} + 1$, the argument of the third term of (10.33) becomes

$$\left[\left(\frac{N_s}{p_1} + 1 \right) \left(\theta + \frac{\pi p_1}{N_s} \right) - \pi - \omega_1 t \right]$$

But this way, it becomes the opposite of the first term argument, so the first step, mmf harmonic $\frac{N_s}{p_1} + 1$, and the first slot opening harmonics subtract each

other. The opposite is true for $v = \frac{N_s}{p_1} - 1$ when they are added. So the slot openings may amplify or attenuate the effect of the step harmonics of order $v = \frac{N_s}{p_1} \mp 1$, respectively.

Other effects such as differential leakage fields affected by slot openings have been investigated in Chapter 6 when the differential leakage inductance has been calculated. Also we have not yet discussed the currents induced by the flux harmonics in the rotor and in the stator conductors.

In what follows, some attention will be paid to the main effects of airgap flux and mmf harmonics: parasitic torque and radial forces.

10.9. PARASITIC TORQUES

Not long after the cage-rotor induction motors reached industrial use, it was discovered that a small change in the number of stator or rotor slots prevented the motor to start from any rotor position or the motor became too noisy to be usable. After Georges (1896), Punga (1912), Krondl, Lund, Heller, Alger, and Jordan presented detailed theories about additional (parasitic) asynchronous torques, Dreyfus (1924) derived the conditions for the manifestation of parasitic synchronous torques.

10.9.1. When do asynchronous parasitic torques occur?

Asynchronous parasitic torques occur when a harmonic v of the stator mmf (or its airgap flux density) produces in the rotor cage currents whose mmf harmonic has the same order v .

The synchronous speed of these harmonics ω_{1v} (in electrical terms) is

$$\omega_{1v} = \frac{\omega_1}{v} \quad (10.34)$$

The stator mmf harmonics have orders like: $v = -5, +7, -11, +13, -17, +19, \dots$, in general, $6c_1 \pm 1$, while the stator slotting introduces harmonics of the order $\frac{c_1 N_s}{p_1} \pm 1$. The higher the harmonic order, the lower its mmf amplitude. The slip

S_v of the v th harmonic is

$$S_v = \frac{\omega_{1v} - \omega_r}{\omega_{1v}} = 1 - v(1 - S) \quad (10.35)$$

For synchronism $S_v = 0$ and, thus, the slip for the synchronism of harmonic v is

$$S = 1 - \frac{1}{v} \quad (10.36)$$

For the first mmf harmonic ($v = -5$), the synchronism occurs at

$$S_5 = 1 - \frac{1}{(-5)} = \frac{6}{5} = 1.2 \quad (10.37)$$

For the seventh harmonic ($v = +7$)

$$S_7 = 1 - \frac{1}{(+7)} = \frac{6}{7} \quad (10.38)$$

All the other mmf harmonics have their synchronism at slips

$$S_5 > S_v > S_7 \quad (10.39)$$

In a first approximation, the slip synchronism for all harmonics is $S \approx 1$. That is close to motor stall, only, the asynchronous parasitic torques occur.

As the same stator current is at the origin of both the stator mmf fundamental and harmonics, the steady state equivalent circuit may be extended in series to include the asynchronous parasitic torques (Figure 10.4).

The mmf harmonics, whose order is lower than the first slot harmonic $v_{smin} = \frac{N_s}{p_1} \pm 1$, are called phase belt harmonics. Their order is: $-5, +7, \dots$. They are all indicated in Figure 10.4 and considered to be mmf harmonics.

One problem is to define the parameters in the equivalent circuit. First of all the magnetizing inductances $X_{m5}, X_{m7}, \dots$ are in fact the “components” of the up to now called differential leakage inductance.

$$L_{mv} = \frac{6\mu_0 \tau L}{\pi^2 p_1 g K_c (1 + K_{st})} \left(\frac{W_1 K_{wv}}{v} \right)^2 \quad (10.40)$$

The saturation coefficient K_{st} in (10.40) refers to the teeth zone only as the harmonics wavelength is smaller than that of the fundamental and therefore the flux paths close within the airgap and the stator and rotor teeth/slot zone.

The slip $S_v \approx 1$ and thus the slip frequency for the harmonics $S_v \omega_1 \approx \omega_1$. Consequently, the rotor cage manifests a notable skin effect towards harmonics, much as for short-circuit conditions. Consequently, $R'_{rv} \approx (R'_r)_{start}$, $L'_{rlv} \approx (L'_{rl})_{start}$.

As these harmonics act around $S = 1$, their torque can be calculated as in a machine with given current $I_s \approx I_{start}$.

$$T_{ev} \approx \frac{3R'_{rstart}}{S_v \omega_1} I_{start}^2 \frac{L_{mv}^2}{\left[(L'_{rl})_{start} + L_{mv} \right]^2 + \left(\frac{R'_{rstart}}{S_v \omega_1} \right)^2} \quad (10.41)$$

In (10.41), the starting current I_{start} has been calculated from the complete equivalent circuit where, in fact, all magnetization harmonic inductances L_{mv}

have been lumped into $X_{sl} = \omega_l L_{sl}$ as a differential inductance as if $S_v = \infty$. The error is not large.

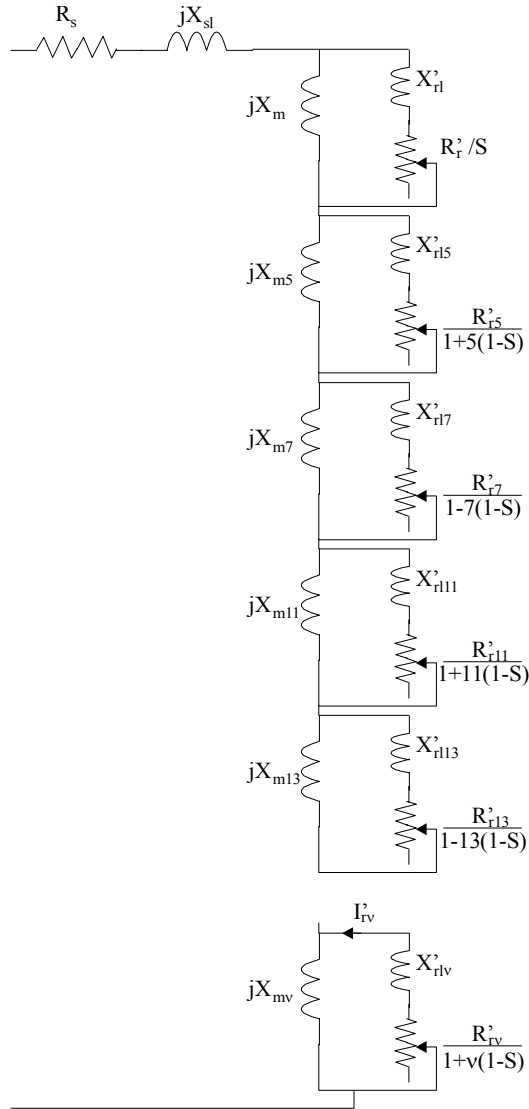


Figure 10.4 Equivalent circuit including asynchronous parasitic torques due to mmf harmonics (phase band and slot driven)

Now (10.41) reflects a torque/speed curve similar to that of the fundamental (Figure 10.5). The difference lies in the rather high current, but factor K_{wv}/v is

rather small and overcompensates this situation, leading to a small harmonic torque in well-designed machines.

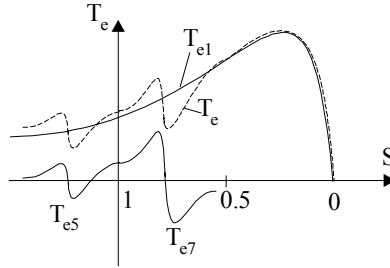


Figure 10.5 Torque/slip curve with 5th and 7th harmonic asynchronous parasitic torques

We may dwell a little on (10.41) noticing that the inductance L_{mv} is of the same order (or smaller) than $(L'_{rl})_{start}$ for some harmonics. In these cases, the coupling between stator and rotor windings is small and so is the parasitic torque.

Only if $(L'_{rl})_{start} < L_{mv}$ a notable parasitic torque is expected. On the other hand, to reduce the first parasitic asynchronous torques T_{e5} or T_{e7} , chording of stator coils is used to make $K_{w5} = 0$.

$$K_{w5} = \frac{\sin \frac{5\pi}{6}}{q \sin \left(\frac{5\pi}{6q} \right)} \sin \frac{5\pi y}{2\tau} \approx 0; \quad \frac{y}{\tau} = \frac{4}{5} \quad (10.42)$$

As $y/\tau = 4/5$ is not feasible for all values of q , the closest values are $y/\tau = 5/6, 7/9, 10/12$, and $12/15$. As can be seen for $q = 5$, the ratio $12/15$ fulfils exactly condition (10.42). In a similar way the 7th harmonic may be cancelled instead, if so desired.

The first slot harmonics torque ($v_{smin} = \frac{N_s}{p_1} \pm 1$) may be cancelled by using skewing.

$$K_{wv_{smin}} = \frac{\sin \frac{v_{smin} \pi}{6}}{q \sin \left(\frac{v_{smin} \pi}{6q} \right)} \sin \frac{v_{smin} \pi y}{2\tau} \cdot \frac{\sin v_{smin} \frac{c}{\tau} \frac{\pi}{2}}{v_{smin} \frac{c}{\tau} \frac{\pi}{2}} \approx 0 \quad (10.43)$$

From (10.43),

$$v_{smin} \frac{c}{\tau} \frac{\pi}{2} = K\pi; \quad \frac{c}{\tau} = \frac{2K}{v_{smin}} = \frac{2K}{\frac{N_s}{p_1} \pm 1} \quad (10.44)$$

In general, the skewing c/τ corresponds to 0.5 to 2 slot pitches of the part (stator or rotor) with less slots.

For $N_s = 36$, $p_1 = 2$, $m = 3$, $q = N_s/2p_1m = 36/(2 \cdot 2 \cdot 3) = 3$: $c/\tau = 2/17(19) \approx 1/9$. This, in fact, means a skewing of one slot pitch as a pole has $q \cdot m = 3 \cdot 3 = 9$ slot pitches.

10.9.2. Synchronous parasitic torques

Synchronous torques occur through the interaction of two harmonics of the same order (one of the stator and one of the rotor) but originating from different stator harmonics. This is not the case, in general, for wound rotor whose harmonics do not adapt to stator ones, but produces about the same spectrum of harmonics for the same number of phases.

For cage-rotors, however, the situation is different. Let us first calculate the synchronous speed of harmonic v_1 of the stator.

$$n_{1v_1} = \frac{f_1}{p_1 v_1} \quad (10.45)$$

The relationship between a stator harmonic v and the rotor harmonic v' produced by it in the rotor is

$$v - v' = c_2 \frac{N_r}{p_1} \quad (10.46)$$

This is easy to accept as it suggests that the difference in the number of periods of the two harmonics is a multiple of the number of rotor slots N_r .

Now the speed of rotor harmonics v' produced by the stator harmonic v with respect to rotor $n_{2vv'}$ is

$$n_{2vv'} = p \frac{S_v f_1}{p_1 v'} = \frac{n_1}{v'} \left[1 - \frac{v}{1-S} \right] \quad (10.47)$$

With respect to stator, $n_{1vv'}$ is

$$n_{1vv'} = n + n_{2vv'} = \frac{f_1}{p_1 v'} [1 + (v - v')(1-S)] \quad (10.48)$$

With (10.46), Equation (10.48) becomes

$$n_{1vv'} = \frac{f_1}{p_1 v'} \left[1 + c_2 \frac{N_r}{p_1} (1-S) \right] \quad (10.49)$$

Synchronous parasitic torques occur when two harmonic field speeds n_{1v_1} and $n_{1vv'}$ are equal to each other.

$$n_{1v_1} = n_{1vv'} \quad (10.50)$$

$$\text{or} \quad \frac{1}{v_1} = \frac{1}{v'} \left[1 + c_2 \frac{N_r}{p_1} (1 - S) \right] \quad (10.51)$$

It should be noticed that v_1 and v' may be either positive (forward) or negative (backward) waves. There are two cases when such torques occur.

$$\begin{aligned} v_1 &= +v' \\ v_1 &= -v' \end{aligned} \quad (10.52)$$

When $v_1 = +v'$, it is mandatory (from (10.51)) to have $S = 1$ or zero speed.

On the other hand, when $v_1 = -v'$,

$$(S)_{v_1=+v'} = 1; \text{ standstill} \quad (10.53)$$

$$(S)_{v_1=-v'} = 1 + \frac{2p_1}{c_2 N_r} \ll 1 \quad (10.54)$$

As seen from (10.54), synchronous torques at nonzero speeds, defined by the slip $(S)_{v_1=-v'}$, occur close to standstill as $N_r \gg 2p_1$.

Example 10.2. Synchronous torques

For the unusual case of a three-phase IM with $N_s = 36$ stator slots and $N_r = 16$ rotor slots, $2p_1 = 4$ poles, frequency $f_1 = 60\text{Hz}$, let us calculate the speed of the first synchronous parasitic torques as a result of the interaction of phase belt and step (slot) harmonics.

Solution

First, the stator and rotor slotting-caused harmonics have the orders

$$v_{1s} = c_1 \frac{N_s}{p_1} + 1; \quad v_s' = c_2 \frac{N_r}{p_1} + 1; \quad c_1 c_2 \ll 0 \quad (10.55)$$

The first stator slot harmonics v_{1s} occur for $c_1 = 1$.

$$v_{1f} = +1 \frac{36}{2} + 1 = 19; \quad v_{1b} = -1 \frac{36}{2} + 1 = -17 \quad (10.56)$$

On the other hand, a harmonic in the rotor $v_f' = +17 = -v_{1b}$ is obtained from (10.55) for $c_2 = 2$.

$$v_f' = +2 \frac{16}{2} + 1 = +17 \quad (10.57)$$

The slip S for which the synchronous torque occurs (10.54),

$$S_{(17)} = 1 + \frac{2 \cdot 2}{2 \cdot 16} = \frac{9}{8}; \quad n_{17} = \frac{f_1}{p_1} (1 - S_{(17)}) = \frac{60}{2} \left(1 - \frac{9}{8} \right) \cdot 60 = -255 \text{rpm} \quad (10.58)$$

Now if we consider the first (phase belt) stator mmf harmonic ($6c_1 \pm 1$), $v_f = +7$ and $v_b = -5$, from (10.55) we discover that for $c_2 = -1$.

$$v_b' = -1 \frac{16}{2} + 1 = -7 \quad (10.59)$$

So the first rotor slot harmonic interacts with the 7th order (phase belt) stator mmf harmonic to produce a synchronous torque at the slip.

$$S_{(7)} = 1 + \frac{2 \cdot 2}{(-1) \cdot 16} = \frac{3}{4}; \quad n_7 = \frac{f_1}{P_1} (1 - S_{(7)}) = \frac{60}{2} \left(1 - \frac{3}{4}\right) \cdot 60 = 450 \text{rpm} \quad (10.60)$$

These two torques occur as superposed on the torque/speed curve, as discontinuities at corresponding speeds and at any other speeds their average values are zero (Figure 10.6).

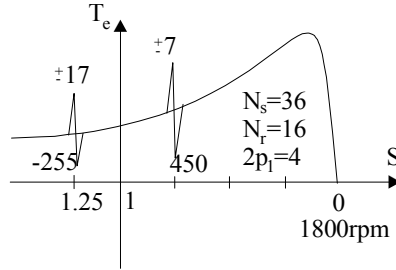


Figure 10.6 Torque/speed curve with parasitic synchronous torques

To avoid synchronous torques, the conditions (10.52) have to be avoided for harmonics orders related to the first rotor slot harmonics (at least) and stator mmf phase belt harmonics.

$$6c_1 + 1 \neq c_2 \frac{N_r}{p_1} + 1 \quad (10.61)$$

$$c_1 = \pm 1, \pm 2 \text{ and } c_2 = \pm 1, \pm 2$$

For stator and rotor slot harmonics

$$c_1 \frac{N_s}{p_1} + 1 \neq c_2 \frac{N_r}{p_1} + 1 \quad (10.62)$$

Large synchronous torque due to slot/slot harmonics may occur for $c_1 = c_2 = \pm 1$ when

$$N_s - N_r = 0; \quad v_1 = v_2' \quad (10.63)$$

$$N_s - N_r = 2p_1; \quad v_1 = -v_2' \quad (10.64)$$

When $N_s = N_r$, the synchronous torque occurs at zero speed and makes the motor less likely to start without hesitation from any rotor position. This situation has to be avoided in any case, so $N_s \neq N_r$.

Also from (10.61), it follows that ($c_1, c_2 > 0$), for zero speed,

$$c_2 N_r \neq 6c_1 p_1 \quad (10.65)$$

Condition (10.64) refers to first slot harmonic synchronous torques occurring at nonzero speed:

$$n_{v_1=v_2'} = \frac{f_1}{p_1} (1-S) = \frac{f_1}{p_1} \frac{2p_1}{c_2 N_r} = \frac{-2f_1}{c_2 N_r} \quad (10.66)$$

In our case, $c_2 = \pm 1$. Equation (10.66) can also be written in stator harmonic terms as

$$n_{v_1=-v_2'} = \frac{f_1}{p_1 v_1} = \frac{f_1}{p_1} \frac{1}{\left(c_1 \frac{Z_1}{p_1} + 1 \right)} = \frac{f_1}{c_1 Z_1 + p_1} \quad (10.67)$$

with $c_1 = \pm 1$.

The synchronous torque occurs at the speed $n_{v_1=-v_2'}$, if there is a geometrical phase shift angle γ_{geom} between stator and rotor mmf harmonics v_1 and $-v_2'$, respectively. The torque should have the expression T_{sv_1} .

$$T_{sv_1} = (T_{sv_1})_{\text{max}} \sin(c_1 Z_1 + p_1) \cdot \gamma_{\text{geom}} \quad (10.68)$$

The presence of synchronous torques may cause motor locking at the respective speed. This latter event may appear more frequently with IMs with descending torque/speed curve (Design C, D), [Figure 10.7](#).

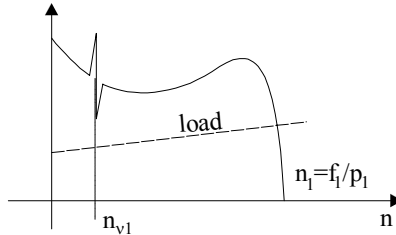


Figure 10.7 Synchronous torque on descending torque/speed curve

So far we considered mmf harmonics and slot harmonics as sources of synchronous torques. However, there are other sources for them such as the airgap magnetic conductance harmonics due to slot leakage saturation.

10.9.3. Leakage saturation influence on synchronous torques

The airgap field produced by the first rotor mmf slot harmonic $v' = \pm \frac{N_r}{p_1} + 1$ due to the airgap magnetic conduction fluctuation caused by leakage saturation (second term in (10.19)) is

$$B_{gv'} = \mu_0 F_{2v_m} \cdot \cos \left[\left(\pm \frac{N_r}{p_1} + 1 \right) \theta \mp (\omega_1 \pm N_r \Omega_r) t \right] \cdot \frac{a_0'}{g} \sin(2\theta - \omega_1 t) \quad (10.69)$$

If $B_{gv'}$ contains harmonic (term) multiplied by $\cos(v\theta \mp \omega_1 t)$ for any $n = \pm 6c_1 + 1$ that corresponds to the stator mmf harmonics, a synchronous torque condition is again obtained. Decomposing (10.69) into two terms we find that only if

$$\frac{N_r}{p_1} \pm 3 \neq (6c_1 \pm 1) \quad (10.70)$$

such torques are avoided. Condition (10.70) becomes

$$N_r \neq 2p_1(3c_1 \mp 1); \quad c_1 = 1, 2, 3, \dots \quad (10.71)$$

The largest such torque occurs in interaction with the first stator slot mmf harmonic $v_1 = \frac{N_s}{p_1} \pm 1$ unless

$$\frac{N_s}{p_1} \pm 1 \neq \frac{N_r}{p_1} \pm 3 \quad (10.72)$$

$$\text{So,} \quad N_r \neq N_s \mp 4p_1 \quad (10.73)$$

As an evidence of the influence of synchronous torque at standstill with rotor bars skewed by one stator slot pitch, some experimental results are given on [Table 10.2](#). [1]

Table 10.2. Starting torque (maximum and minimum value)

Motor power (kW) $f_1 = 50\text{Hz}$ $2p_1 = 2$ poles	N_s	N_r	Starting torque (Nm)	
			max	min
0.55	24	18	1.788	0
0.85	24	22	0.75	0.635
1.4	24	26	0.847	0.776
2	24	28	2	1.7647
3	24	30	5.929	2.306

As [Table 10.2](#) shows the skewing of rotor bars cannot prevent the occurrence of synchronous torque and a small change in the number of rotor

slots drastically changes the average starting torque. The starting torque variation (with rotor position) is a clear indication of synchronous torque presence at stall.

The worst situation occurs with $N_r = 18 = 6c_1p_1$ (10.65). The speed for this synchronous torque comes from (10.69) as

$$\pm 3\omega_1 + N_r\Omega_r = \pm\omega_1 \quad (10.74)$$

or

$$\Omega_r = \pm \frac{4\omega}{N_r}; \quad n = \frac{4f_1}{N_r} \quad (10.75)$$

Calculating the synchronous torques (even at zero speed) analytically is a difficult task. It may be performed by calculating the stator mmf harmonics airgap field interaction with the pertinent rotor current mmf harmonic.

After [1], the maximum tangential force $f_{\theta\max}$ per unit rotor area due to synchronous torques at standstill should be

$$f_{\theta\max} \approx \frac{\mu_0 (n_s I_s \sqrt{2})^2 d}{4\gamma_{\text{slot}}} \quad (10.76)$$

where $n_s I_s \sqrt{2}$ is the peak slot mmf, γ_{slot} is the stator slot angle, and d is the greatest common divisor of N_s and N_r .

From this point of view, the smaller d , the better. The number $d = 1$ if N_s and N_r are prime numbers and it is maximum for $N_s = N_r = d$.

With $N_s = 6p_1q_1$, for q integer, that is an even number with N_r an odd number, with $d = 1$ the smallest variation with rotor position (safest start) of starting torque is obtained.

However, with N_r an odd number, the IM tends to be noisy, so most IMs have an even number of rotor slots, but with $d = 2$ if possible.

10.9.4. The secondary armature reaction

So far in discussing the synchronous torques, we considered only the reaction of rotor currents on the fields of the fundamental stator current. However, in some cases, harmonic rotor currents can induce in the stator windings additional currents of a frequency different from the power grid frequency. These additional stator currents may influence the IM starting properties and add to the losses in an IM. [4,5] These currents are related to the term of secondary armature reaction. [4]

With delta stator connection, the secondary armature reaction currents in the primary windings find a good path to flow, especially for multiple of 3 order harmonics. Parallel paths in the stator winding also favor this phenomenon by its circulating currents.

For wound (slip ring) rotor IMs, the field harmonics are not influenced by delta connection or by the parallel path. [6] Let us consider the first step (slot) stator harmonic $\frac{N_s}{p_1} \pm 1$ whose current $I_1 \left(\frac{N_s \pm 1}{p_1} \right)$ produces a rotor current $I_2 \left(\frac{N_s \pm 1}{p_1} \right)$.

$$I_2 \left(\frac{N_s \pm 1}{p_1} \right) \approx \frac{I_1 \left(\frac{N_s \pm 1}{p_1} \right)}{1 + \tau_d \left(\frac{N_s \pm 1}{p_1} \right)} \quad (10.77)$$

τ_d is the differential leakage coefficient for the rotor cage. [1]

$$\tau_d \left(\frac{N_s \pm 1}{p_1} \right) = \left[\frac{\pi(N_s \pm p_1)}{N_r} \right]^2 \cdot \frac{1}{\sin^2 \left[\frac{\pi(N_s \pm p_1)}{N_r} \right]} - 1 \quad (10.78)$$

As known, N_s and N_r are numbers rather close to each other, so $\tau_d \left(\frac{N_s \pm 1}{p_1} \right)$ might be a large number and thus the rotor harmonic current is small. A strong secondary armature reaction may change this situation drastically. To demonstrate this, let us notice that $I_2 \left(\frac{N_s \pm 1}{p_1} \right)$ can now create harmonics of order v'

(10.46), for $c_2 = 1$ and $v = \left(\frac{N_s}{p_1} \pm 1 \right)$:

$$v' = \frac{N_r}{p_1} \pm \left(\frac{N_s}{p_1} \pm 1 \right) \quad (10.79)$$

The v' rotor harmonic mmf $F_{v'}$ has the amplitude:

$$F_{v'} = F_2 \left(\frac{N_s \pm 1}{p_1} \right) \cdot \frac{N_s \pm p_1}{N_r \pm N_s \pm p_1} \quad (10.80)$$

If these harmonics induce notable currents in the stator windings, they will, in fact, reduce the differential leakage coefficient from $\tau_d \left(\frac{N_s \pm 1}{p_1} \right)$ to $\tau'_d \left(\frac{N_s \pm 1}{p_1} \right)$. [1]

$$\tau'_d \left(\frac{N_s \pm 1}{p_1} \right) \approx \tau_d \left(\frac{N_s \pm 1}{p_1} \right) - \left(\frac{N_s \pm p_1}{N_r - (N_s \pm p_1)} \right)^2 \quad (10.81)$$

Now the rotor current $I_{2\left(\frac{N_s \pm 1}{p_1}\right)}$ of (10.77) will become $I'_{2\left(\frac{N_s \pm 1}{p_1}\right)}$.

$$I'_{2\left(\frac{N_s \pm 1}{p_1}\right)} = \frac{I_{1\left(\frac{N_s \pm 1}{p_1}\right)}}{\tau'_d\left(\frac{N_s \pm 1}{p_1}\right)} \quad (10.82)$$

The increase of rotor current $I'_{2\left(\frac{N_s \pm 1}{p_1}\right)}$ is expected to increase both asynchronous and synchronous torques related to this harmonic.

In a delta connection, such a situation takes place. For a delta connection, the rotor current (mmf) harmonics

$$v' = \frac{N_r}{p_1} - \left(\frac{N_s}{p_1} \pm 1 \right) = 3c \quad c = 1, 3, 5, \dots \quad (10.83)$$

can flow freely because their induced voltages in the three stator phases are in phase (multiple of 3 harmonics). They are likely to produce notable secondary armature reaction.

To avoid this undesirable phenomenon,

$$|N_s - N_r| \neq (3c \mp 1)p_1 = 2p_1, 4p_1, \dots \quad (10.84)$$

An induction motor with $2p_1 = 6$, $N_s = 36$, $N_r = 28$ slots does not satisfy (10.84), so it is prone to a less favourable torque-speed curve in Δ connection than it is in star connection.

In parallel path stator windings, circulating harmonics current may be responsible for notable secondary armature reaction with star or delta winding connection.

These circulating currents may be avoided if all parallel paths of the stator winding are at any moment at the same position with respect to rotor slotting.

However, for an even number of current paths, the stator current harmonics of the order $\frac{N_r}{p_1} - \frac{N_s}{p_1} - 1$ or $\frac{N_r}{p_1} - \frac{N_s}{p_1} + 1$ may close within the winding if the two numbers are multiples of each other. The simplest case occurs if

$$|N_s - N_r| = 3p_1 \quad (10.85)$$

When (10.85) is fulfilled, care must be exercised in using parallel path stator windings.

We should also notice again that the magnetic saturation of stator and rotor teeth lead to a further reduction of differential leakage by 40 to 70%.

10.9.5. Notable differences between theoretical and experimental torque/speed curves

By now the industry has accumulated enormous data on the torque/speed curves of IM of all power ranges.

In general, it was noticed that there are notable differences between theory and tests especially in the braking regime ($S > 1$). A substantial rise of torque in the braking regime is noticed. In contrast, a smaller reduction of torque in the large slip motoring regime appears frequently.

A similar effect is produced by transverse (lamination) rotor currents between skewed rotor bars.

However, in this latter case, there should be no difference between slip ring and cage rotor, which is not the case in practice.

Finally these discrepancies between theory and practice occur even for insulated copper bar cage rotors where the transverse (lamination) rotor currents cannot occur (Figure 10.8).

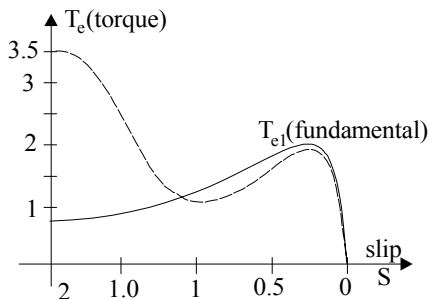


Figure 10.8 Torque-slip curve (relative values)
for $N_s = 36$, $N_r = 28$, $2p_1 = 4$, skewed rotor (one slot pitch), insulated copper bars

What may cause such a large departure of experiments from theory? Although a general, simple, answer to this question is still not available, it seems intuitive to assume that the airgap magnetic conductance harmonics due to slot openings and the tooth head saturation due leakage path saturation are the main causes of spectacular braking torque augmentation and notable reduction high slip motoring torque in.

This is true when the contact resistance of rotor bars (practically insulated rotor bars) is high and, thus, the transverse (lamination) currents in skewed rotors are small.

Slot openings have been shown to increase considerably the parasitic torques. (Remember we showed earlier in this chapter that the first slot (step) stator mmf harmonic $v_{smin} = \frac{N_s}{p_1} - 1$ is amplified by the influence of the first airgap magnetic conductance—due to slot openings – (10.33)).

Now the leakage path saturation produced airgap magnetic conductance (10.11) may add further to amplify the parasitic torques.

For skewed rotors, at high slips, where parasitic torques occur, main path flux saturation due to the uncompensated (skewing) rotor mmf may lead to a fundamental torque increase in the braking region (due to higher current, for lower leakage inductance) and to the amplification of parasitic torque through main path saturation-caused airgap conductance harmonics (10.20). And, finally, rotor eccentricity may introduce its input to this braking torque escalation.

We have to stress again that leakage flux path saturation and skin effects alone, also cause important increases in rotor resistance, decreases in motor reactance, and thus produce also more fundamental torque.

In fact, the extreme case on [Figure 10.8](#) with a steady increase of torque with slip (for $S > 1$) seems to be a mixture of such intricate contributions. However, the minimum torque around zero speed shows the importance of parasitic torques as the skin effect and leakage saturation depend on $\sqrt{S}$; so they change slower with slip than [Figure 10.7](#) suggests.

10.9.6. A case study: $N_s/N_r = 36/28$, $2p_1 = 4$, $y/\tau = 1$ and $7/9$; $m = 3$ [7]

As there is no general answer, we will summarize the case of a three-phase induction motor with $P_n = 11$ kW, $N_s/N_r = 36/28$, $2p_1 = 4$, $y/\tau = 9/9$ and $7/9$ (skewed rotor by one stator slot pitch).

For this machine, and $y/\tau = 9/9$, with straight slots, condition (10.71) is not satisfied as

$$N_r = 28 = 2p_1(3c \mp 1) = 2 \cdot 2(3c + 1), \text{ for } c = 2 \quad (10.86)$$

This means that a synchronous torque occurs at the speed

$$n = \frac{4f_1}{N_r} = \frac{4 \cdot 50 \cdot 60}{28} = 215 \text{ rpm} \quad (10.87)$$

Other synchronous, but smaller, torques may be detected. A very large synchronous torque has been measured at this speed (more than 14 Nm, larger than the peak torque for 27% voltage). This leads to a negative total torque at 215 rpm; successful acceleration to full speed is not always guaranteed. It is also shown that as the voltage increases and saturation takes place, the synchronous torque does not increase significantly over this value.

Also, a motor of 70 kW, $N_s/N_r = 48/42$, $y/\tau = 10/12$ with Δ connection and 4 parallel current paths has been investigated. As expected, a strong synchronous torque has been spotted at standstill because condition (10.65) is not met as

$$N_r = 6 \frac{c_1}{c_2} p_1 = 6 \frac{7}{2} \cdot 2 = 42 \quad (10.88)$$

The synchronous torque at stall is amplified by Δ connection with 4 paths in parallel stator winding as $N_s - N_r = 48 - 42 = 6 = 3p_1 = 3 \cdot 2$.

The situation is so severe that, for 44% voltage and 30% load torque, the motor remains blocked at zero speed.

Strong variations of motor torque with position have been measured and calculated satisfactorily with the finite difference method. Saturation consideration seems mandatory for realistic torque calculations during starting transients.

It has also been shown that the amplification of parasitic torque due to slot openings is higher for $N_r > N_s$ because the differential leakage coefficient $\tau_d \left(\frac{N_{s \pm 1}}{p_1} \right)$ is smaller in this case, allowing for large rotor mmf harmonics. From

the point of view of parasitic torques, the rotor with $N_r < N_s$ is recommended and may be used without skewing for series-connected stator windings.

For $N_r > N_s$ skewing is recommended unless magnetic wedges in the stator slots are used.

10.9.7. Evaluation of parasitic torques by tests (after [1])

To assist potential designers of IMs, here is a characterization of torque/speed curve for quite a few stator /rotor slot numbers of practical interest.

For $N_s = 24$ stator slots, $f_1 = 50$ Hz, $2p_1 = 4$

- $N_r = 10$; small synchronous torques at $n = 150, 300, 600$ rpm and a large one at $n = -300$ rpm
- $N_r = 16$; unsuitable (high synchronous torque at $n = 375$ rpm)
- $N_r = 18$; very suitable (low asynchronous torque dips at $n = 155$ rpm and at -137 rpm)
- $N_r = 19$; unsuitable because of high noise, although only small asynchronous torque dips occur at $n = 155$ rpm and $n = -137$ rpm
- $N_r = 20$; greatest synchronous torque of all situations investigated here at $n = -300$ rpm, unsuitable for reversible operation.
- $N_r = 22$; slight synchronous torques at $n = 690, 600, \pm 550, 214, -136$ rpm
- $N_r = 24$; unsuitable because of high synchronous torque at $n = 0$ rpm
- $N_r = 26$; larger asynchronous torque deeps at $n = 115, -136$ rpm and synchronous one at $n = -300$ rpm
- $N_r = 27$; high noise
- $N_r = 28$; high synchronous torque dip at $n = 214$ rpm and slight torque dips at $n = 60, -300, -430$ rpm. Asynchronous torque dips at $n = 115, -136$ rpm. With skewing, the situation gets better, but at the expense of 20% reduction in fundamental torque.
- $N_r = 30$; typical asynchronous torque dips at $n = 115, -136$ rpm, small synchronous ones at $n = 600, \pm 400, \pm 300, 214, \pm 200, -65, -100, -128$ rpm. Practically noiseless.
- $N_r = 31$; very noisy.

Number of slots $N_s = 36$, $f_1 = 50$ Hz, $2p_1 = 4$

This time, the asynchronous torques are small. At $n = 79$ to 90 rpm, they become noticeable for higher number of rotor slots (44, 48).

- $N_r = 16$; large synchronous torque at $n = -188, +376$ rpm
- $N_r = 20$; synchronous torque at $n = -376$ rpm
- $N_r = 24$; large synchronous torque at $n = 0$ rpm, impractical
- $N_r = 28$; large synchronous torque at $n = -214$ rpm, $-150, -430$ rpm
- $N_r = 31$; good torque curve but very noisy
- $N_r = 32$; large synchronous torque at $n = -188$ rpm and a smaller one at $n = -375$ rpm;
- $N_r = 36$; impractical, motor locks at zero speed;
- $N_r = 44$; large synchronous torque at $n = -136$ rpm, smaller ones at $n = 275, -40, -300$ rpm
- $N_r = 48$; moderate synchronous torque dip at zero speed

Number of slots $N_s = 48$, $f_1 = 50$ Hz, $2p_1 = 4$

- $N_r = 16$; large synchronous torque at $n = -375$, a smaller one at $n = -188$ rpm
- $N_r = 20$; large synchronous torque at $n = -300$ rpm, small ones at $n = 600, 375, 150, 60, -65, -136, -700$ rpm
- $N_r = 22$; large synchronous torque dip at $n = -136$ rpm due to the second rotor slip harmonic ($c_2 = 2$, $n = \frac{2f_1}{c_2 N_r} = \frac{-2 \cdot 50 \cdot 60}{2 \cdot 22} = -136$ rpm)
- $N_r = 28$; high synchronous torque at $n = 214$ rpm, smaller ones at $n = 375, 150, -65, -105, -136, -214$ rpm
- $N_r = 32$; high synchronous torque at $n = -188$ rpm, small ones at $n = 375, 188, -90, 90, -300$ rpm
- $N_r = 36$; large torque dip at zero speed, not good start
- $N_r = 44$; very large synchronous torque at $n = -136$ rpm, smaller ones at $n = 275, 214, 60, -300$ rpm
- $N_r = 48$; motor locks at zero speed ($N_s = N_r = 48!$), after warming up, however, it starts!

10.10. RADIAL FORCES AND ELECTROMAGNETIC NOISE

Tangential forces in IMs are producing torques which smoothly drive the IM (or cause torsional vibrations) while radial forces are causing vibration and noise.

This kind of vibration and noise is produced by electromagnetic forces which vary along the rotor periphery and with time. Using Maxwell tensor, after neglecting the tangential field component in the airgap (as it is much lower than the radial one), the radial stress on the stator is

$$p_r = \frac{B^2(\theta_m, t)}{2\mu_0}; \quad \theta = \theta_m p_1 \quad (10.89)$$

where $B(\theta, t)$ is the resultant airgap flux density.

The expression of $B(\theta_m, t)$ is, as derived earlier in this chapter,

$$B(\theta_m, t) = \frac{\mu_0}{g} (F_1(\theta_m, t) + F_2(\theta_m, t)) \cdot \lambda(\theta_m, t) \quad (10.90)$$

with $F_1(\theta_m, t)$ and $F_2(\theta_m, t)$ the stator and rotor mmf and $\lambda(\theta_m, t)$ the airgap magnetic conductance as influenced by stator and rotor slot openings, leakage, and main path saturation and eccentricity.

$$\lambda(\theta, t) = \lambda_0 + \sum_{\sigma=1}^{\infty} \lambda_{\sigma} \cos(\sigma\theta_m + \varphi_{\sigma}) + \sum_{\rho=1}^{\infty} \lambda_{\rho} \cos\left(\rho\left(\theta_m - \frac{\omega_r t}{p_1}\right) + \varphi_{\rho}\right) + \dots \quad (10.91)$$

$$F_1(\theta_m, t) = \sum F_{1v} \cos(v\theta_m \pm \omega_v t) \quad (10.92)$$

$$F_2(\theta_m, t) = \sum F_{2\mu} \cos(\mu\theta_m \pm \omega_{\mu} t + \varphi_{\mu}) \quad (10.93)$$

In (10.89 through 10.93) we have used the mechanical angle θ_m as the permeance harmonics are directly related to it, and not the electrical angle $\theta = p_1\theta_m$. Consequently, now the fundamental, with p_1 pole pairs, will be $v_1 = p_1$ and not $v = 1$.

Making use of (10.92 and 10.93) in (10.89) we obtain terms of the form

$$p_r = A_r \cos(\gamma\theta_m - \Omega_r t) \quad (10.94)$$

$\gamma = 0, 1, 2, \dots$ The frequency of these forces is $f_r = \frac{\Omega_r}{2p_1}$, which is not be

confused to the rotor speed ω_r/p_1 .

If the number of pole pairs (γ) is small, stator vibrations may occur due to such forces. As inferred from (10.90), there are terms from same harmonic squared and interaction terms where 1(2) mmf harmonics interact with an airgap magnetic conductance. Such combinations are likely to yield low orders for γ .

For $\gamma = 0$, the stress does not vary along rotor periphery but varies in time producing variable radial pressure on the stator (and rotor).

For $\gamma = 1$ (10.94) yields a single-sided pull on the rotor which circulates with the angular velocity of Ω_r . If Ω_r corresponds to a mechanical resonance of the machine, notable vibration will occur.

Case $\gamma = 1$ results from the interaction of two mechanical harmonics whose order differs by unity.

For $\gamma = 2, 3, 4, \dots$ stator deflections occur. In general, all terms p_r with $\gamma \leq 4$ should be avoided, by adequately choosing the stator and rotor slot number pairs N_s and N_r .

However, as mechanical resonance of stators (without and with frames attached) decreases with motor power increase, it means that a certain N_s, N_r combination may lead to low noise in low power motors but to larger noise in large power motors.

Now as in (10.90), the mmf step harmonics and the airgap magnetic conductance occur, we will treat their influence separately to yield some favourable N_s, N_r combinations.

10.10.1. Constant airgap (no slotting, no eccentricity)

In this case $\lambda(\theta_{m,t}) = \lambda_0$ and a typical p_r term occurs from two different stator and rotor mmf harmonics.

$$(p_r)_{\gamma=v-\mu} = \frac{\mu_0 F_{1v} F_{2\mu}}{2g^2} \lambda_0^2 \cos[(\gamma \mp \mu)\theta_m \pm (\omega_v + \omega_\mu) - \phi_\mu] \quad (10.95)$$

In this case $\gamma = v - \mu$, that is the difference between stator and rotor mmf mechanical harmonics order.

Now if we are considering the first slot (step) mmf harmonics v, μ ,

$$\begin{aligned} v &= N_s \pm p_1 \\ \mu &= N_r \pm p_1 \end{aligned} \quad (10.96)$$

It follows that to avoid such forces with $\gamma = 0, 1, 3, 4$,

$$\begin{aligned} |N_s - N_r| &\neq 0, 1, 2, 3, 4 \\ |N_s - N_r| &\neq 2p_1, 2p_1 \pm 1, 2p_1 \pm 2, 2p_1 \pm 3, 2p_1 \pm 4 \end{aligned} \quad (10.97)$$

The time variation of all stator space harmonics has the frequency $\omega_v = \omega_1 = 2\pi f_1$, while for the rotor,

$$\omega_\mu = \omega_1 \mp c N_r \omega_r = \omega \left[1 \mp c \frac{N_r}{p_1} (1 - S) \right] \quad (10.98)$$

Notice that ω_r is the rotor mechanical speed here.

Consequently the frequency of such forces is $\omega_v \pm \omega_\mu$.

$$\omega_\xi = \omega_v \pm \omega_\mu = \begin{cases} \omega_1 c \frac{N_r}{p_1} (1 - S) \\ 2\omega_1 + \omega_1 c \frac{N_r}{p_1} (1 - S) \end{cases} \quad (10.99)$$

In general, only frequencies ω_ξ low enough ($c = 1$) can cause notable stator vibrations.

The frequency ω_ξ is twice the power grid frequency or zero at zero speed ($S = 1$) and increases with speed.

10.10.2. Influence of stator/rotor slot openings, airgap deflection and saturation

As shown earlier in this chapter, the slot openings introduce harmonics in the airgap magnetic conductance which have the same order as the mmf slot (step) harmonics.

Zero order ($\gamma = 0$) components in radial stress p_r may be obtained when two harmonics of the same order, but different frequency, interact. One springs from the mmf harmonics and one from the airgap magnetic conductance harmonics. To avoid such a situation simply,

$$\begin{aligned} N_s \pm p_1 \neq N_r \\ N_r \pm p_1 \neq N_s \quad \text{or} \quad |N_s - N_r| \neq \pm p_1 \end{aligned} \quad (10.100)$$

Also, to avoid first order stresses for $\gamma = 2$,

$$2(N_r - N_s \mp p_1) \neq 2 \quad (10.101)$$

$$|N_s - N_r| \neq \pm p_1 \pm 1 \quad (10.102)$$

The frequency of such stresses is still calculated from (10.100) as

$$\omega_\xi = \left[\omega_1 \pm \omega_1 \frac{N_r}{p_1} (1 - S) \right] 2 \quad (10.103)$$

The factor 2 above comes from the fact that this stress component comes from a term squared so all the arguments (of space and time) are doubled.

It has been shown that airgap deformation itself, due to such forces, may affect the radial stresses of low order.

In the most serious case, the airgap magnetic conductance becomes

$$\begin{aligned} \lambda(\theta_m, t) = \frac{1}{g(\theta_m, t)} = [a_0 + a_1 \cos N_s \theta_m + a_2 \cos N_r (\theta_m - \omega_r t)] \cdot \\ \cdot [1 + B \cos(K\theta_m - \Omega t)] \end{aligned} \quad (10.104)$$

The last factor is related to airgap deflection. After decomposition of $\lambda(\theta_m, t)$ in simple sin(cos) terms for $K = p_1$, strong airgap magnetic conductance terms may interact with the mmf harmonics of orders p_1 , $N_s \pm p_1$, $N_r \pm p_1$ again. So we have to have

$$\begin{aligned} (N_s \pm p_1) - (N_r \pm p_1) \neq p_1 \\ \text{or} \quad |N_s - N_r| \neq p_1 \\ \text{and} \quad |N_s - N_r| \neq 3p_1 \end{aligned} \quad (10.115)$$

In a similar way, slot leakage flux path saturation, already presented earlier in this chapter with its contribution to the airgap magnetic conductance, yields conditions as above to avoid low order radial forces.

10.10.3. Influence of rotor eccentricity on noise

We have shown earlier in this chapter that rotor eccentricity—static and dynamic—introduces a two-pole geometrical harmonics in the airgap magnetic conductance. This leads to a $p_1 \pm 1$ order flux density strong components (10.90) in interaction with the mmf fundamentals. These forces vary at twice stator frequency $2f_1$, which is rather low for resonance conditions. In interaction with another harmonic of same order but different frequency, zero order ($\gamma = 0$) vibrations may occur again. Consider the interference with first mmf slot harmonics $N_s \pm p_1$, $N_r \pm p_1$

$$\begin{aligned} (N_s \pm p_1) - (N_r \pm p_1) &\neq p_1 \pm 1 \\ \text{or } |N_s - N_r| &\neq 3p_1 \pm 1 \\ |N_s - N_r| &\neq p_1 \pm 1 \end{aligned} \quad (10.106)$$

These criteria, however, have been found earlier in this paragraph. The dynamic eccentricity has been also shown to produce first order forces ($\gamma = 1$) from the interaction of $p_1 \pm 1$ airgap magnetic conductance harmonic with the fundamental mmf (p_1) as their frequencies is different. Noise is thus produced by the dynamic eccentricity.

10.10.4. Parallel stator windings

Parallel path stator windings are used for large power, low voltage machines (now favored in power electronics adjustable drives). As expected, if consecutive poles are making current paths and some magnetic asymmetry occurs, a two-pole geometric harmonic occurs. This harmonic in the airgap magnetic conductance can produce, at least in interaction with the fundamental mmf, radial stresses with the order $\gamma = p_1 \pm 1$ which might cause vibration and noise.

On the contrary, with two parallel paths, if north poles make one path and south poles the other current path, with two layer full pitch windings, no asymmetry will occur. However, in single layer winding, this is not the case and stator harmonics of low order are produced ($v = N_s/2c$). They will never lead to low order radial stress, because, in general, N_s and N_r are not far from each other in actual motors.

Parallel windings with chorded double layer windings, however, due to circulating current, lead to additional mmf harmonics and new rules to eliminate noise, [1]

$$\begin{aligned}
|N_s - N_r| &= \left| \frac{p_1}{a} \pm p_1 \pm 1 \right|; \quad a - \text{odd} \\
|N_s - N_r| &= \left| \frac{2p_1}{a} \pm p_1 \pm 1 \right|; \quad a - \text{even}
\end{aligned} \tag{10.107}$$

where a is the number of current paths.

Finally, for large power machines, the first slot (step) harmonic order $N_s \pm p_1$ is too large so its phase belt harmonics $n = (6c_1 \pm 1)p_1$ are to be considered in (10.117), [1]

$$\begin{aligned}
\left| N_s \pm \frac{p_1}{a} - (6c_1 \pm 1)p_1 \right| &\neq 1; \quad a > 2, \text{ odd} \\
\left| N_r \pm \frac{2p_1}{a} - (6c_1 \pm 1)p_1 \right| &\neq 1; \quad a > 2, \text{ even} \\
|N_r \pm 2p_1 - (6c_1 \pm 1)p_1| &\neq 1; \quad a = 2
\end{aligned} \tag{10.108}$$

with c_1 a small number $c_1 = 1, 2, \dots$

Symmetric (integer q) chorded windings with parallel paths introduce, due to circulating current, a $2p_1$ pole mmf harmonic which produces effects that may be avoided by fulfilling (10.108).

10.10.5. Slip-ring induction motors

The mmf harmonics in the rotor mmf are now $v = c_2 N_r \pm p_1$ only. Also, for integer q_2 (the practical case), the number of slots is $N_r = 2 \cdot 3 \cdot p_1 \cdot q_2$. This time the situation of $N_s = N_r$ is to be avoided; the other conditions are automatically fulfilled due to this constraint on N_r . Using the same rationale as for the cage rotor, the radial stress order, for constant airgap, is

$$r = v = \mu = (6c_1 \pm 1)p_1 - (6c_2 \pm 1)p_1 \tag{10.109}$$

Therefore γ may only be an even number.

Now as soon as eccentricity occurs, the absence of armature reaction leads to higher radial uncompensated forces than the cage rotors. The one for $\gamma = p_1 \pm 1$ —that in interaction with the mmf fundamental, which produces a radial force at $2f_1$ frequency—may be objectionable because it is large (no armature reaction) and produces vibrations anyway.

So far, the radial forces have been analyzed in the absence of magnetic saturation and during steady state. The complexity of this subject makes experiments in this domain mandatory.

Thorough experiments [8] for a few motors with cage and wound rotors confirm the above analysis in general, and reveal the role of magnetic saturation of teeth and yokes in reducing these forces (above 70 to 80% voltage). The radial forces due to eccentricity level off or even decrease.

During steady state, the unbalanced magnetic pull due to eccentricity has been experimentally proved much higher for the wound rotor than for the cage rotor! During starting transients, both are, however, only slightly higher than for the wound rotor under steady state. Parallel windings have been proven to reduce the uncompensated magnetic pull force ($\gamma = 0$).

It seems that even with 20% eccentricity, UMP may surpass the cage rotor mass during transients and the wound rotor mass even under steady state. [8]

The effect of radial forces on the IM depends not only on their amplitude and frequency but also on the mechanical resonance frequencies of the stator, so the situation is different for low medium power and large power motors.

10.10.6 Mechanical resonance stator frequencies

The stator ring resonance frequency for $r = 0$ (first order) radial forces F_0 is [1]

$$F_0 = \frac{8 \cdot 10^2}{R_a} \sqrt{\frac{G_{yoke}}{G_{yoke} + G_{teeth}}} \quad (10.110)$$

with R_a —average stator yoke radius, G_{yoke} and G_{teeth} stator yoke and teeth weight.

For $r \neq 0$,

$$F_r = F_0 \frac{1}{2\sqrt{3}} \frac{h_{yoke}}{R_a} \frac{r(r^2 - 1)}{\sqrt{r^2 + 1}} K_r \quad (10.111)$$

h_{yoke} —yoke radial thickness.

$K_r = 1$ for $h_{yoke}/R_a > 0.1$ and $0.66 < K_r < 1$ for $h_{yoke}/R_a < 0.1$.

The actual machine has a rather elastic frame fixture to ground and a less than rigid attachment of stator laminations stack to the frame.

This leads to more than one resonance frequency for a given radial force of order r . A rigid framing and fixture will reduce this resonance “multiplication” effect.

To avoid noise, we have to avoid radial force frequencies to be equal to resonance frequencies. For the step harmonics of mmfs contribution (10.100) applies.

for $r = 0$, at $S = 0$ (zeroslip),

$$\frac{F_0}{f_1} \neq c \frac{N_r}{p_1} \quad \text{or} \quad \frac{F_0}{f_1} \neq c \frac{N_r}{p_1} \pm 2; \quad c = 1, 2 \quad (10.122)$$

We may proceed, for $r = 1, 2, 3$, by using F_r instead of F_0 in (10.112) and with $c = 1$.

In a similar way, we should proceed for the frequency of other radial forces (derived in previous paragraphs).

In small power machines, such conditions are easy to meet as the stator resonance frequencies are higher than most of radial stress frequencies. So

radial forces of small pole pairs $r = 0, 1, 2$ are to be checked. For large power motors radial forces with $r = 3, 4$ are more important to account for.

The subject of airgap flux distribution, torque pulsations (or parasitic torques), and radial forces has been approached recently by FEM [9,10] with good results, but at the price of prohibitive computation time.

Consequently, it seems that intuitive studies, based on harmonics analysis, are to be used in the preliminary design study and, once the number of stator/rotor slots are defined, FEM is to be used for precision calculation of parasitic torques and of radial forces.

10.11. SUMMARY

- The fundamental component of resultant airgap flux density distribution in interaction with the fundamental of stator (or rotor) mmf produces the fundamental (working) electromagnetic torque in the IM.
- The placement of windings in slots (even in infinitely thin slots) leads to a stepped-like waveform of stator (rotor) mmfs which exhibit space harmonics besides the fundamental wave (with p_1 pole pairs). These are called mmf harmonics (or step harmonics); their lower orders are called phase-belt harmonics $v = 5, 7, 11, 13, \dots$ to the first “slot” harmonic order

$$v_{smin} = \frac{N_s}{p_1} \pm 1. \text{ In general, } v = (6c_1 \pm 1).$$

- A wound rotor mmf has its own harmonic content $\mu = (6c_2 \pm 1)$.
- A cage rotor, however, adapts its mmf harmonic content (μ) to that of the stator such that

$$v - \mu = \frac{c_2 N_r}{p_1}$$

- v and μ are electric harmonics, so $v = \mu = 1$ means the fundamental (working) wave.
- The mechanical harmonics v_m, μ_m are obtained if we multiply v and μ by the number of pole pairs,

$$v_m = p_1 v; \quad \mu_m = p_1 \mu;$$

- In the study of parasitic torques, it seems better to use electric harmonics, while, for radial forces, mechanical harmonics are favored. As the literature uses both concepts, we thought it appropriate to use them both in a single chapter.
- The second source of airgap field distribution is the magnetic specific airgap conductance (inversed airgap function) $\lambda_{1,2}(\theta_m, t)$ variation with stator or/and rotor position. In fact, quite a few phenomena are accounted for by decomposing $\lambda_{1,2}(\theta_m, t)$ into a continuous component and various harmonics. They are outlined below.

- The slot openings, both on the stator and rotor, introduce harmonics in $\lambda_{1,2}(\theta_m, t)$ of the orders $c_1 N_s / p_1$, $c_2 N_r / p_1$ and $(c_1 N_s \pm c_2 N_r) p_1$ with $c_1 = c_2 = 1$ for the first harmonics.
- At high currents, the teeth heads saturate due to large leakage fluxes through slot necks. This effect is similar to a fictitious increase of slot openings, variable with slot position with respect to stator (or rotor) mmf maximum. A $\lambda_{1,2}(\theta_m, t)$ second order harmonic ($4p_1$ poles) traveling at the frequency of the fundamental occurs mainly due to this aspect.
- The main flux path saturation produces a similar effect, but its second order harmonic ($4p_1$ poles) is in phase with the magnetization mmf and not with stator or rotor mmfs separately!
- The second permeance harmonic also leads to a third harmonic in the airgap flux density.
- The rotor eccentricity, static and dynamic, produces mainly a two-pole harmonic in the airgap conductance. For a two-pole machine, in interaction with the fundamental mmf, a homopolar flux density is produced. This flux is closing axially through the frame, bearings, and shaft, producing an a.c. shaft voltage and bearing current of frequency f_1 (for static eccentricity) and Sf_1 (for the dynamic eccentricity).
- The interaction between mmf and airgap magnetic conductance harmonics is producing a multitude of harmonics in the airgap flux distribution.
- Stator and rotor mmf harmonics of same order and same stator origin produce asynchronous parasitic torques. Practically all stator mmf harmonics produce asynchronous torques in cage rotors as the latter adapts to the stator mmf harmonics. The no-load speed of asynchronous torque is $\omega_r = \omega_1 / v$.
- As $v \geq 5$ in symmetric (integer q) stator windings, the slip S for all asynchronous torques is close to unity. So they all occur around standstill.
- The rotor cage, as a short-circuited multiphase winding, may attenuate asynchronous torques.
- Chording is used to reduce the first asynchronous parasitic torques (for $v = -5$ or $v = +7$); skewing is used to cancel the first slot mmf harmonic $v = 6q_1 \pm 1$. (q_1 —slots/pole/phase).
- Synchronous torques are produced at some constant specific speeds where two harmonics of same order $v_1 = \pm v'$ but of different origin interact.
- Synchronous torques may occur at standstill if $v_1 = +v'$ and at

$$S = 1 + \frac{2p_1}{c_2 N_r}; \quad -1 \geq C_2 \geq 1$$

for $v_1 = -v'$.

- Various airgap magnetic conductance harmonics and stator and rotor mmf harmonics may interact in a cage rotor IM many ways to produce synchronous torques. Many stator/rotor slot number N_s/N_r combinations are to be avoided to eliminate most important synchronous torque. The main

benefit is that the machine will not lock into such speed and will accelerate quickly to the pertinent load speed.

- The harmonic current induced in the rotor cage by various sources may induce certain current harmonics in the stator, especially for Δ connection or for parallel stator windings ($a > 1$).
- This phenomenon, called secondary armature reaction, reduces the differential leakage coefficient τ_d of the first slot harmonic $v_{smin} = N_s/p_1 \pm 1$ and thus, in fact, increases its corresponding (originating) rotor current. Such an augmentation may lead to the amplification of some synchronous torques.
- The stator harmonics currents circulating between phases (in Δ connection) or in between current paths (in parallel windings), whose order is multiples of three may be avoided by forbidding some N_s, N_r combinations ($N_s - N_r \neq 2p_1, 4p_1, \dots$).
- Also, if the stator current paths are in the same position with respect to rotor slotting, no such circulating currents occur.
- Notable differences, between the linear theory and tests have been encountered with large torque amplifications in the braking region ($S > 1$). The main cause of this phenomenon seems to be magnetic saturation.
- Slot opening presence also tends to amplify synchronous torques. This tendency is smaller if $N_r < N_s$; even straight rotor slots (no skewing) may be adopted. Attention to noise for no skewing!
- As a result of numerous investigations, theoretical and experimental, clear recommendations of safe stator/rotor slot combinations are now given in some design books. Attention is to be paid to the fact that, when noise is concerned, low power machines and large power machines behave differently.
- Radial forces are somehow easier to calculate directly by Maxwell's stress method from various airgap flux harmonics.
- Radial stress (force per unit stator area) p_r is a wave with a certain order r and a certain electrical frequency Ω_r . Only $r = 0, 1, 2, 3, 4$ cases are important.
- Investigating again the numerous combination contributions to the mechanical stress components coming from the mmfs and airgap magnetic conductance harmonics, new stator/rotor slot number N_s/N_r restrictions are developed.
- Slip ring IMs behave differently as they have clear cut stator and rotor mmf harmonics. Also damping of some radial stress component through induced rotor current is absent in wound rotors.
- Especially radial forces due to rotor eccentricity are much larger in slip ring rotor than in cage rotor with identical stators, during steady state. The eccentricity radial stress during starting transients are however about the same.

- The circulating current of parallel winding might, in this case, reduce some radial stresses. By increasing the rotor current, they reduce the resultant flux density in the airgap which, squared, produces the radial stress.
- The effect of radial stress in terms of stator vibration amplitude manifests itself predominantly with zero, first, and second order stress harmonics ($r = 0, 1, 2$).
- For large power machines, the mechanical resonance frequency is smaller and thus the higher order radial stress $r = 3, 4$ are to be avoided if low noise machines are to be built.
- For precision calculation of parasitic torques and radial forces, after preliminary design rules have been applied, FEM is to be used, though at the price of still very large computation time.
- Vibration and noise is a field in itself with a rich literature and standardization. [11,12]

10.12. REFERENCES

1. B. Heller, V. Hamata, Harmonics Effects in Induction Motors, Chapter 6, Elsevier, 1977.
2. R. Richter, Electric Machines, Second edition, Vol.1, pp.173, Verlag Birkhäuser, Basel, 1951 (in German).
3. K. Vogt, Electric Machines—Design of Rotary Electric Motors, Chapter 10, VEB Verlag Technik, Berlin, 1988. (in German)
4. K. Oberretl, New Knowledge on Parasitic Torques in Cage Rotor Induction Motors, Bulletin Oerlikon 348, 1962, pp.130 – 155.
5. K. Oberretl, The Theory of Harmonic Fields of Induction Motors Considering the Influence of Rotor Currents on Additional Stator Harmonic Currents in Windings with Parallel Paths, Archiv für Electrotechnik, Vol.49, 1965, pp.343 – 364 (in German).
6. K. Oberretl, Field Harmonics Theory of Slip Ring Motor Taking Multiple Reaction into Account, Proc of IEE, No.8, 1970, pp.1667 – 1674.
7. K. Oberretl, Parasitic Synchronous and Pendulation Torques in Induction Machines; the Influence of Transients and Saturation, Part II – III, Archiv für Elek. Vol.77, 1994, pp.1 – 11, pp.277 – 288 (in German).
8. D.G. Dorrel, Experimental Behaviour of Unbalanced Magnetic Pull in 3-phase Induction Motors with Eccentric Rotors and the Relationship with Teeth Saturation, IEEE Trans Vol. EC – 14, No.3, 1999, pp.304 – 309.
9. J.F. Bangura, N.A. Dermerdash, Simulation of Inverter-fed Induction Motor Drives with Pulse-width-modulation by a Time-stepping FEM Model—Flux Linkage-based Space Model, IEEE Trans, Vol. EC – 14, No.3, 1999, pp.518 – 525.
10. A. Arkkio, O. Lingrea, Unbalanced Magnetic Pull in a High Speed Induction Motor with an Eccentric Rotor, Record of ICEM – 1994, Paris, France, Vol.1, pp.53 – 58.
11. S.J. Yang, A.J. Ellison, Machinery Noise Measurement, Clarendon Press, Oxford, 1985.

12. P.L. Timar, Noise and Vibration of Electrical Machines, Technical Publishers, Budapest, Hungary, 1986.